Approximating Instantaneous Rates of Change

1. A toy car begins moving from rest. Let $s(t)$ represent the distance a toy car has moved away from its initial position (in feet) and let t represent the number of seconds elapsed since the toy car started moving. Values of $s(t)$ for various values of t are provided in the table below.

t	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6
$s(t)$	36	38	41	45	47	50	52	55	56	59	61

   (a) Which is the best approximation of the speed of the toy car 3 seconds after it started moving and why?

      i. $\dfrac{50-45}{.2} = 25$ ft/sec

      ii. $\dfrac{50-47}{.1} = 30$ ft/sec

      iii. $\dfrac{3.1-2.9}{5} = .04$ ft/sec

      iv. $\dfrac{3.1-3}{3} \approx .03333$ ft/sec

   (b) Find the best possible overestimate and underestimate of the speed of the toy car 2.8 seconds after it started moving. What is the greatest possible error between your speed estimates and the true speed of the car at 2.8 seconds after it started moving?

   (c) Are either of $\dfrac{59-56}{.1} = 30$ ft/sec or $\dfrac{61-59}{.1} = 20$ ft /sec a better estimate for the speed of the toy car at 3.5 seconds than the other? Explain why or why not.

   (d) Approximate the acceleration of the toy car 2.8 seconds after it started moving.

2. A waiter is pouring water into a cylindrical cup of radius 2 inches at a decreasing pour rate. When there are $6\pi in^3$ of water in the cup, the height of the water is 1.5 inches. When there are $16\pi in^3$ of water in the cup, the height of the water is 4 inches.

   (a) Is it possible to determine the instantaneous rate of change between the amount of water in the cup and the height of the water in the cup when there are $10.5378\pi in^3$ of water in the cup with this information? Why or why not?

   (b) If you feel more information is needed, use the formula for the volume of a cylinder to generate more average rate of change estimates.

3. Let Δt be a change in the time away from $t = 3$ seconds (either forward or backward in time). A ball is thrown into the air with an initial (vertical) velocity of 12m/s from an initial height of 2m. A simplified formula to determine the height of the ball in the air (without air resistance) is $h(t) = -9.8t^2 + 12t + 2$ meters. Calculate the average vertical speed of the ball between $t = 3 + \Delta t$ (Δt can be negative) and $t = 3$ seconds.

   (a) Simplify the average speed into two terms, one term depending on Δt and one term not depending on Δt.

   (b) Determine the range of possible speeds for the ball at $t = 3$ seconds for small values of Δt. If you keep choosing smaller values of Δt, what happens to the range of possible values for the ball's speed at $t = 3$ seconds?