# Optimization: Algebraic Modeling

1. Suppose the function $g(t) = 6t^2 - 0.75t^3$ represents the height of a golfball above the ground (in yards) $t$ seconds since the ball was struck (only for values of $t$ between 0 and 8). By solving which of the following equations can we determine the value of $t$ for which the height of the ball above the ground is maximized?

   (a) $g'(t) = 12t - 2.25t^2$

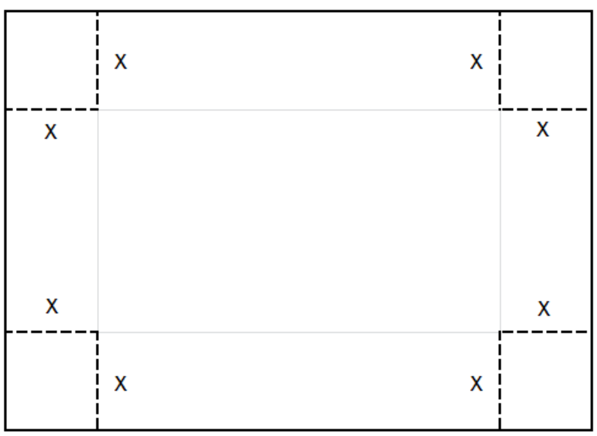
   (b) $0 = 12t - 2.25t^2$

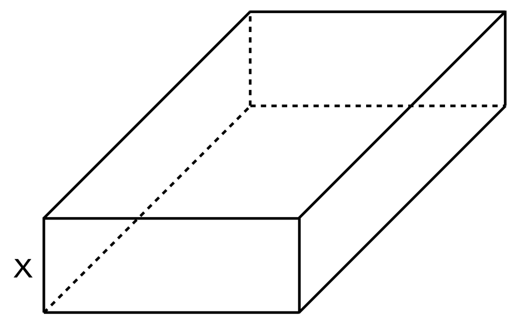
   (c) $g(t) = 6t^2 - .75t^3$

   (d) $0 = 6t^2 - .75t^3$

   (e) $g''(t) = 12 - 4.5t$

2. You create a box with no top (shown on the right) by cutting squares of side length $x$ out of a $9 \times 24$-inch piece of cardboard (shown on the left) and folding up the sides.

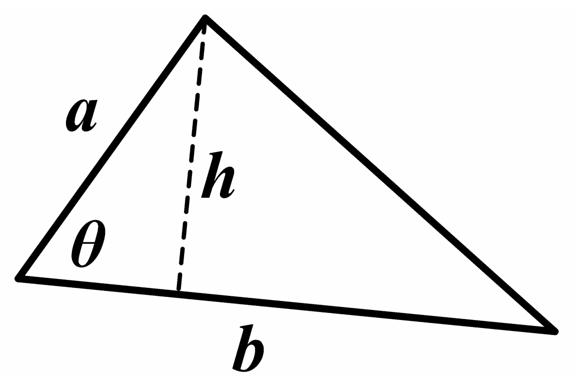


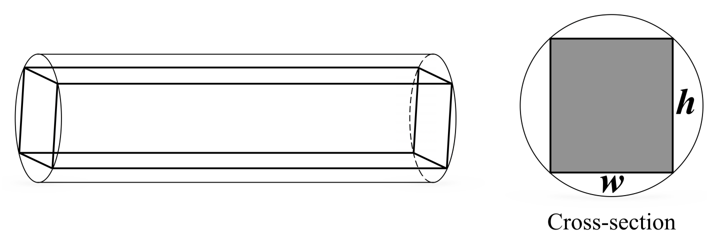

   (a) Define a function $V$ that gives the volume of the box in terms of side length $x$ of the square cutout.

   (b) Use calculus to determine the value of $x$ for which the box has maximal volume. **Justify your answer**.

3. One side of a triangle has length $a$ and the hypotenuse has length $b$. Use calculus to prove that the triangle's area is maximized if it is a right triangle. Use the diagram below to support your reasoning.




4. Suppose a rectangular beam is cut from a cylindrical log of diameter 40 cm as shown in the image below. The strength of a beam, $S$, is given by the formula $S = 7wh^2$, where $w$ represents the width of the beam in centimeters and $h$ represents the height of the beam in centimeters. Find the width and height of the beam with maximum strength that can be cut from the log. Justify your answer.




5. A rectangular swimming pool is to be built with an area of 1,800 square feet. The owner wants 5-foot wide decks along two sides of the pool and 10-foot wide decks at the two ends as shown in the picture below. Find the dimensions of the smallest possible combined area of the pool and deck satisfying these conditions. **Justify your response.**